THE ASTROPHYSICAL JOURNAL, 543:L1–L4, 2000 November 1

THE INTERMITTENT BEHAVIOR OF THE COSMIC MASS FIELD REVEALED BY A QSO'S Lyα FOREST

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Received 2000 June 26; accepted 2000 August 16; published 2000 October 6

ABSTRACT

The intermittent behavior of the space-scale distribution of the Lyα-transmitted flux of QSO HS 1700+64 has been analyzed via a discrete wavelet transform. We found that there are two strong indications of intermittency on scales down to about 10 h^{-1} kpc: (1) the probability distribution function of the local fluctuations of the flux has a significantly long tail on small scales, and (2) the local power spectrum of the flux shows prominent spiky structures on small scales. Moreover, the local power spectrum, averaged on regions with different sizes, shows similar spiky structures. Therefore, the random mass density field traced by the Lyα forest is rougher on smaller scales, consistent with singular clustering.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The structures of the cosmic mass field on scales from the submegaparsec to the kiloparsec have attracted a lot of attention recently. High-resolution *N*-body simulations show that the core profiles of massive halos of the cold dark matter (CDM) cosmogony are singular (Navarro, Frenk, & White 1997; Moore et al. 2000; Jing & Suto 2000), while the halo profiles required by the rotation curves of dwarf galaxies (Flores & Primack 1994; Burkert 1995) are shallower than the numerical results. The central cusps of dark halos also differ from the soft halo profiles inferred from low surface brightness galaxies (de Blok & McGaugh 1997). The cores of galaxies and clusters are even found to be consistent with the thermal equilibrium model with a "universal" mass density (Firmani et al. 2000). In other words, the singular behavior of cosmic clustering has not been detected in the cores of galaxies and clusters.

We can now ask the following question: can the singular behavior, if it exists, be revealed by methods other than the mass profile of galaxies and clusters? At first glance, this goal seems unachievable since singular mass density profiles can only be seen in cores of galaxies and clusters. However, a random mass field $\rho(x)$, consisting of rare singular structures randomly scattered in the low-mass density background, is typically intermittent (Zeldovich, Ruzmaikin, & Sokoloff 1990). A basic characteristic of an intermittent field is that the density *difference* between two neighboring positions, $|\rho(x+r)-\rho(x)|$, can be "abnormally" large when r is very small. That is, the rare events of large density difference $|\rho(x+r)-\rho(x)|$ on small scales r have a higher probability than that for a Gaussian field. Such singular behavior of a random mass field means that the probability density function (PDF) of the density differences on small scales r has a long tail. Obviously, the effects of a long-tailed PDF of the density difference are not limited to the singular mass profile. This motivated us to look for the PDF's long tail and its effects by using samples other than the cores of galaxies and clusters.

The PDF's long tail has not yet been seriously studied. The most popular statistical measure of large-scale structures—the power spectrum of a mass field—is insensitive to the PDF's long tail. Furthermore, the density difference, $|\rho(x+r)-\rho(x)|$, is a quantity localized in space x and on scale r, and so a space-scale decomposition is necessary. Thus, it is impossible to identify the effects of a long-tailed PDF of the density difference by using the power spectrum or any statistic that is not based on proper space-scale decomposition.

In this Letter, using a discrete wavelet transform (DWT), we look for the long-tailed effects from a Lyα forest QSO. It is believed that, point by point, the distribution of baryonic diffuse matter is almost proportional to the underlying dark matter density. Moreover, the optical depth of Lyα is linearly dependent on the baryonic density. Therefore, the high-resolution data of the transmitted flux of a QSO's absorption would be a good candidate for revealing the long-tailed PDF of the cosmic mass field on small scales.

2. METHOD

Let us consider a one-dimensional random mass density field $\rho(x)$ in the spatial range L. With a DWT space-scale decomposition, the local density difference, $|\rho(x+r)-\rho(x)|$, is represented by the wavelet function coefficients (WFCs) as

$$\tilde{\epsilon}_{j,l} = \langle \psi_{j,l}, \rho \rangle, \tag{1}$$

where $\psi_{j,l}(x)$ is the orthonormal and complete basis of the DWT, and $\langle \dots \rangle = \int \dots dx$ is the inner product (Daubechies 1992). We use the DAUB4 wavelet (Press et al. 1993) for our analysis throughout this Letter.[3] The WFC $\tilde{\epsilon}_{j,l}$ is the density fluctuation on the scale $L/2^j$ at the position $l = 0, \dots, 2^j - 1$, or the mean density difference between nearest neighbors on the scale $L/2^j$ at l. If the "fair sample hypothesis" (Peebles 1980) holds, then the 2^j values of $\tilde{\epsilon}_{j,l}$ form an ensemble of the density differences on scale j, and therefore the distribution of $\tilde{\epsilon}_{j,l}$ is a reasonable estimate of the PDF of the density differences on scale j (Fang & Thews 1998).

The second-order statistics $|\tilde{\epsilon}_{j,l}|^2$ describes the power of the perturbations of the mode (j, l). In other words, at a given

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[3] See also O. M. Nielsen 1998, Wavelet in Scientific Computing (http://www.imm.dtu.dk/˜omni).

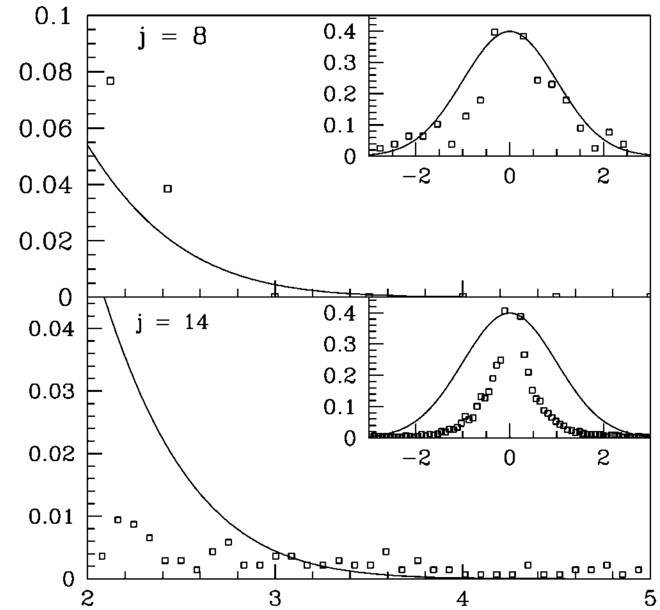

FIG. 1.—The PDFs of the WFCs $\tilde{\epsilon}_{j,l}$ for $j = 8$ and $j = 14$ are shown in the top right-hand corner of each panel. The tail is shown in the main part of each panel. The horizontal axis is for $\tilde{\epsilon}_{j,l}/\sigma$, where σ is the variance of the sample. The vertical axis is the probability density. The solid curve is the Gaussian distribution with zero mean and unit variance. The PDF of $j = 14$ at zero is ∼4.5, so it is not shown in the panels.

position l, the local power spectrum is given by

$$P_{j,l} = \tilde{\epsilon}^2_{j,l}. \quad (2)$$

By averaging $P_{j,l}$ over all positions l, we have

$$P_j = \frac{1}{2^j}\sum_{l=0}^{2^j-1} |\tilde{\epsilon}_{j,l}|^2 = \frac{1}{2^j}\sum_{l=0}^{2^j-1} P_{j,l}. \quad (3)$$

It has been shown that P_j actually is a band-averaged Fourier power spectrum (Pando & Fang 1998; Fang & Feng 2000).

The Fourier power spectrum lacks phase information, and therefore P_j cannot show the phase-related features of clustering. However, $P_{j,l}$ is phase-sensitive. One can search for the phase-related features of the mass field by the *local* DWT power spectrum $P_{j,l}$.

We can generalize the definition of the local DWT power spectrum (eq. [2]) as follows. First, we chop L into 2^{j_s} subintervals, labeled $l_s = 0, 1, \ldots (2^{j_s} - 1)$. Each subinterval has a length $L/2^{j_s}$. Then the local DWT power spectrum at subinterval l_s is given by

$$P_{j,\{j_s,l_s\}} = \frac{1}{2^{j-j_s}} \sum_{l=l_s 2^{j-j_s}}^{(l_s+1)2^{j-j_s}-1} |\tilde{\epsilon}_{j,l}|^2, \quad (4)$$

which is the power on scale $L/2^j$ localized on l_s with size $L/2^{j_s}$.

For a Gaussian field, the local power spectrum $P_{j,l}$ will not show structures with respect to l. On the other hand, the singular behavior of a random field is measured by the exponent α defined by $|\rho(x + r) - \rho(x)| \sim r^{\alpha}$. The larger the α is, the smoother the field on small scales is and vice versa. If the exponent α is negative, there is an actual singularity of the field. Therefore, the singular behavior can be revealed by the roughness of the local power spectrum on small scales. The WFC local power spectrum can also measure the index n of the power-law profile $\rho \sim r^{-n}$ for an individual core.

3. SAMPLE AND ANALYSIS

The sample used for the analysis is the Lyα-transmitted flux of QSO HS 1700+64. This sample has been employed to study the evolution of structure (Bi & Davidsen 1997) and the Fourier and DWT power spectra (Feng & Fang 2000). The recovered power spectrum has been found to be consistent with the CDM model on scales larger than about 0.1 h^{-1} Mpc. The data ranges from 3727.012 to 5523.554 Å, with a resolution of 3 km s^{-1}, for a total of 55,882 pixels. In this Letter, we use the first 25,000 pixels for analysis, which correspond to $z = 2.07$–2.65 or λ = 3727.012–4434.266 Å. On average, a pixel is about 0.029 Å, which is equivalent to a physical size of ∼5 h^{-1} kpc at $z \sim 2$ for a flat universe. We pad 7768 null pixels at the end to utilize a fast wavelet transform algorithm, which requires the data size in powers of 2. It does not affect the analysis because the wavelet transform is localized. Moreover, we subject DWTs directly to pixels without transforming them into physical positions. The relation between pixel number and physical position is not linear, but it does not affect structures on small scales. Thus, we ignore the effect of the nonlinear relation in our present analysis.

Most lines in the Lyα-transmitted flux are due to absorptions by gases in cool and low-density regions. The pressure gradients are generally less than gravitational forces. That is, the gas, and hence the transmitted flux, should be a good tracer of the dark matter. Nevertheless, small-scale structures of the dark matter field may be smoothed out by the velocity dispersion of Lyα forest gases. Therefore, to identify the clustering feature, we will statistically compare the real data with their phase-randomized counterpart, which is obtained by taking the inverse transform of the Fourier coefficients of the original data after randomizing their phases uniformly over $[0, 2\pi]$ without changing their amplitudes.

3.1. *The PDFs of WFCs*

In Figure 1, we show the PDFs of the WFCs for $j = 8$ and 14. Each PDF is normalized to have unit variance. For the scale $j = 8$, the departure from the Gaussian distribution is not so significant. In particular, no tail shows in the $j = 8$ PDF, i.e., no WFCs found to be $\tilde{\epsilon}_{j,l} \geq 3\,\sigma$. For $j = 14$, the PDF of a WFC has 2^{14} events. Therefore, if the PDF were Gaussian, the number of events larger than 3 σ would be about 44. However, the data show 234 events beyond the 3 σ range. Furthermore, a Gaussian PDF predicts that the number of events larger than 5 σ should be 0.01, while the data show more than 100 events larger than 5 σ. The data extend to beyond 15 σ on both sides. Therefore, the PDF has a significantly long tail on small scales. In other words, the field is rougher on smaller scales. This indicates that the field may contain singular structures.

The shapes of the two PDFs of $j = 8$ and 14 are very different from each other. That is, the two stochastic variables $\tilde{\epsilon}_{j,l}$ and $j = 8$, 14 do not relate to each other as

$$\tilde{\epsilon}_{j,l} = 2^{\beta(j-j')}\tilde{\epsilon}_{j',l}, \quad (5)$$

where β is a constant. Therefore, the mass field traced by the QSO HS 1700+64 is unlikely to be self-similar.

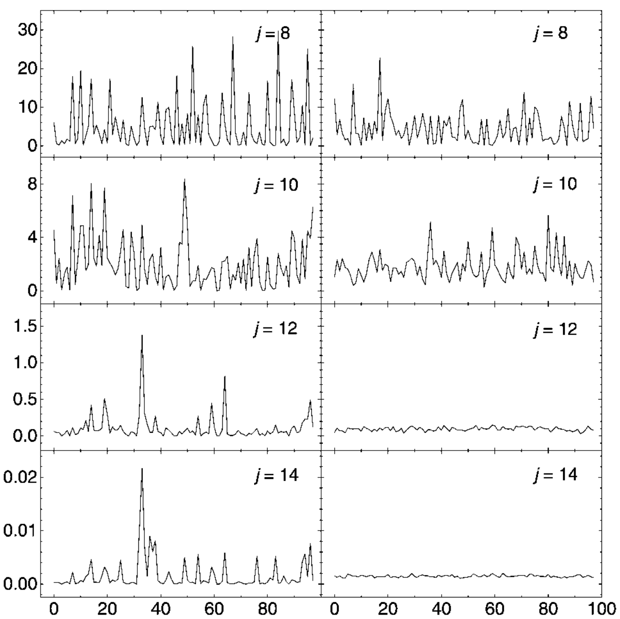

FIG. 2.—The vertical axis represents $2^{j-j_s}P_{j,\{j_s,l_s\}}$ with $j_s = 7$. The left- and right-hand panels are for the HS 1700+64–transmitted flux and its phase-randomized counterpart, respectively. The horizontal axis represents the position l_s in units of $2^{15-j} \times 5\ h^{-1}$ kpc. The scales of the local power spectra, $j = 8$, 10, 12, and 14, correspond to physical scales of $2^{15-j} \times 5\ h^{-1}$ kpc.

3.2. *Local Power Spectrum*

In Figure 2, we plot the local DWT power spectra of the HS 1700+64–transmitted flux and its phase-randomized counterpart. We take $j_s = 7$; i.e., we chop the entire sample into 128 subintervals and, in each subinterval, calculate the power spectra for $j = 8$–14 that correspond to physical scales of $2^{15-j} \times 5\ h^{-1}$ kpc.

Figure 2 shows that the $j = 8$ local power spectrum for real data is not very different from its phase-randomized counterpart. This is consistent with the $j = 8$ PDF shown in Figure 1. It is closer to Gaussian. On the other hand, the $j = 12$–14 local spectra are very rough, showing remarkably spiky structures that completely disappear in the phase-randomized counterpart. The spiky features mean that a significant part of the power is concentrated in some small areas. This feature is a result of the long-tailed PDF of the density difference; i.e., there is a higher probability of "abnormal" density change. The smaller the scale, the more pronounced the spiky features. This again points to a singular clustering of the cosmic mass field.

It should be emphasized that the spikes in the local power spectrum with high j do *not* always correspond to the peaks in the density distribution (or the absorption lines in the optical spectrum). The WFCs $\tilde{\epsilon}_{j,l}$ describe the *difference* in density between intervals of length $2^{15-j} \times 5\ h^{-1}$ Mpc. The average of $\tilde{\epsilon}_{j,l}$ over l is generally zero. The mean power (or variance) at $j = 14$ is $P_{14} = 1.2 \times 10^{-5}$. Thus, even a single event that is greater than 10 σ at $j = 14$ does not always refer to high density, and it can happen in regions other than high-density cores. The spikes denote the positions where the density (or the absorption optical depth) undergoes a dramatic change, which is the key indicator of the singular behavior of a random field.

The wavelet functions $\psi_{j,l}$ are orthogonal, and therefore the local power spectrum on scale j does not contaminate perturbations on other scales. This is very different from the density distribution smoothed by a window function on scale j. The peaks identified from the window-function–smoothed density field contain all contributions from perturbations on scales $j' \leq j$. Therefore, the peaks in a smoothed field are actually given by a superposition of perturbations on large and small scales. They may not show singular features because the PDFs of large-scale (or $j' < j$) perturbations are closer to Gaussian.

We should estimate the possible distortion of the long-tail effects caused by the velocity dispersion of gases. We calculated the $j = 14$ local power spectrum with subinterval $j_s = 14$, which is plotted in Figure 3. This local power spectrum has almost the same spiky features as those identified in the $j = 14$, $j_s = 7$ spectrum (see Fig. 2). That is, most of the spikes shown by the power localized in the subinterval with a size of about 600 h^{-1} kpc actually are localized in the subinterval with a size of only about few 10 h^{-1} kpc. This result indicates that the contamination of the gas velocity dispersion may not be significant, at least, for prominent spikes. Thus, the spiky structures should come mainly from the underlying mass field.

Intermittency can more clearly be seen in Figure 3. The mean powers (eq. [3]) of the real data (*left panel*) and the randomized counterpart (*right panel*) of the $j_s = 14$, $j = 14$ local spectrum actually are the same, while the spikes of the real data are higher than the mean power by a factor of a few tens (even hundreds). That is, in the real sample, most power of the $j = 14$ perturbations is concentrated in the spikes, and almost no power, i.e., $P_{j,\{j_s,l_s\}} \simeq 0$, is concentrated in places other than the spikes. This is a typical intermittent distribution.

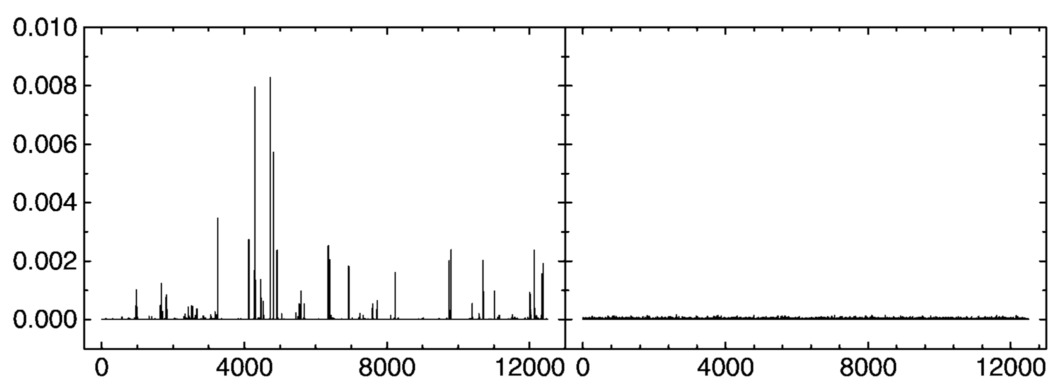

FIG. 3.—Same as Fig. 2 with $j = 14$, but taking $j_s = 14$

4. CONCLUSIONS

With the PDFs of $\tilde{\epsilon}_{j,l}$ and the local DWT power spectra, we show that the mass field traced by the Lyα forest QSO HS 1700+64 is neither Gaussian nor self-similar but is intermittent. The spiky features shown in the local DWT power spectrum are remarkably pronounced on scales down to about 10 h^{-1} kpc. Moreover, the long tail and the spiky features are substantial on smaller scales. This indicates that the cosmic mass field is rougher on smaller scales, which is consistent with singular clustering.

A big advantage of intermittency is that one can detect singular clustering using the statistical features of an entire random density field and not be limited to the cores of galaxies and clusters. The information on intermittency extracted from Lyα forests would be important for testing models of cosmic clustering in terms of their singular behavior.

We thank D. Tytler for kindly providing the data on the Keck spectrum of HS 1700+64. P. J. would also like to thank Robert Maier for his help. H. Z. thanks David Burstein for helpful discussions.

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